



# A coordination approach to the essentiality of money<sup>☆</sup>



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## ABSTRACT

The essentiality of money is commonly justified on efficiency grounds, i.e., money achieves socially desirable allocations which could not be achieved by alternative technologies of exchange. In this paper we argue that what makes money achieve such allocations is its ability to overcome coordination frictions. Intuitively, the fact that money is a permanent record of past production implies that an agent is willing to produce in exchange for money even if he believes that many of his future partners will not accept money.

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## 1. Introduction

Monetary theorists justify the importance of money on the fact that money allows to achieve socially desirable allocations. An implication of this view is that a model where money is essential requires limitations on agents' abilities to monitor each other.<sup>1</sup> It is still an open question through the extent of the limitations on monitoring which are necessary to render money relevant. For instance, in order to ensure that money is essential, models of money that serve as workhorse to much work on monetary theory, such as [Kiyotaki and Wright \(1993\)](#), [Trejos and Wright \(1995\)](#), [Shi \(1995\)](#), and [Lagos and Wright \(2005\)](#), simply assume that agents cannot monitor each other.

In this paper, we propose an alternative rationale for the importance of money. We argue that money is essential because it is better suited to overcome coordination frictions. In a nutshell, we consider a decentralized economy where autarky is the only equilibrium outcome in the absence of money or monitoring, the latter being modeled as a technology which allows an agent to observe the most recent action of his current partner. We first show that money is inessential if we assume away the problem of coordinating in the use of a medium of exchange. We then show that if we are explicit

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<sup>1</sup> [Kocherlakota \(1998\)](#) shows that any monetary allocation can be replicated by a monitoring technology which keeps track of past actions. [Kocherlakota and Wallace \(1998\)](#), and [Cavalcanti and Wallace \(1999\)](#) obtain that money is essential if monitoring is sufficiently limited. A relatively recent trend of papers based on [Lagos and Wright \(2005\)](#) combines money and monitoring. Examples are [Berentsen et al. \(2008\)](#), [Telyukova and Wright \(2008\)](#), [Sanchez and Williamson \(2010\)](#), and [Williamson \(2012\)](#).

about this coordination friction, there exists a region of parameters where agents always coordinate in the use of money but can never coordinate in the use of monitoring.

We are interested in the following question: which technology of exchange, money or monitoring, is less likely to be disrupted by a coordination failure? To deal with this question, we follow the literature on dynamic coordination games and relax the assumption that agents are always free to form any belief as to what other agents will do. We do so by assuming that the economy experiences different states over time and there are remote states where the fundamentals of the economy are such that an agent's optimal action does not depend on his belief about the behavior of other agents.<sup>2</sup> We first consider the economy with money but without monitoring and assume the existence of remote states where an agent strictly prefers to accept money and remote states where he strictly prefers not to accept money. We then consider the economy with monitoring but without money and assume there are remote states where an agent strictly prefers to produce to his partner and remote states where he strictly prefers not to produce. The idea is to use the existence of the remote states to pin down rationalizable outcomes in states which are not remote.

We first show that, irrespective of the technology of exchange, there is a unique rationalizable outcome: outside the remote states, either exchange always takes place, or it never takes place. Whichever outcome emerges depends on the fundamentals of the economy. More interestingly, we show that there exists a region of parameters where exchange takes place under money but it does not take place under monitoring. The broader message we want to convey is that the need to coordinate may render money-like objects essential purely for coordination reasons. That is, the resilience of money may be simply due to the fact that it is relatively easy to coordinate in its use.

The intuition for our result can be summarized as follows. In the presence of coordination frictions, the decision of an agent on whether to produce depends on his belief as to when he will receive the fruits of his current effort. A key parameter impacts this decision: the ability of the agent to keep a record of his current effort. The benefit of money as a coordination device is that, since money is a permanent record of past production, an agent is willing to produce in exchange for money even if he believes that many of his future partners will not accept money. In contrast, the benefit of a monitoring technology which only keeps track of recent actions critically depends on the belief that the fruits of effort will be readily available. This limited monitoring works perfectly and renders money inessential if we take as given that all other agents will always coordinate in the use of the monitoring technology. However, monitoring can be dominated by money if we are explicit about the coordination problems involved in the use of a particular technology of exchange.

Our paper is closely related to [Araujo and Guimaraes \(2014\)](#), which also consider coordination in the use of money in a decentralized economy along the lines of the economy considered here. We complement their paper by introducing monitoring. Our paper is also related to [Camera and Casari \(2014\)](#) and [Duffy and Puzello \(2014\)](#). The experimental evidence in these papers emphasizes the role of money as a coordinating device, since money is used even if it is dominated by social norms of cooperation such as gift-exchange.

The paper is organized as follows. In the next section, we present a benchmark environment where money is inessential. In section 3, we extend the environment and characterize the rationalizable outcomes under monitoring and under money. Section 4 compares monitoring and money and section 5 concludes.

## 2. Benchmark environment

Time is discrete and the economy is populated by a unit continuum of agents with a common discount factor  $\beta \in (0, 1)$ . To motivate a need for exchange, we assume that an agent likes the good produced by any other agent but does not like the good he produces himself. We introduce trade frictions by assuming that agents meet in pairs and at most one good can be produced in a meeting. For tractability, we let goods be indivisible, with the utility of consumption given by  $u$ , and the cost of production given by  $c$ , with  $\beta u > c$ .

At any point in time an agent is either passive or active. Passive agents make no choice while active agents choose between committing ( $C$ ) and not committing ( $\bar{C}$ ) to produce if called upon to do so. Active agents randomly meet passive agents. The status of an agent as passive or active depends on the technology of exchange. We consider two alternative technologies: monitoring and money.

**Monitoring** There exists a technology which allows an active agent to observe the record of his partner, i.e., a label  $g$  (good) or  $b$  (bad), which reflects the latter's action  $C$  (respectively,  $\bar{C}$ ) in the previous period. An agent is passive if he has a record, and he is active if he has no record. In a meeting, the outcome and the future label of an agent depends on the action of the active agent and on the record of the passive agent. Formally, the outcome is determined by a function  $f_c : \{C, \bar{C}\} \times \{g, b\} \rightarrow \{P, \bar{P}\}$ , where  $P$  denotes production and  $\bar{P}$  denotes no production; while the future label of the active agent is determined by a function  $\tau_c : \{C, \bar{C}\} \times \{g, b\} \rightarrow \{g, b\}$ . A monitoring mechanism is a pair of functions  $f_c$  and  $\tau_c$  such that: (i)  $f_c(C, g) = P$  and  $f_c(\bar{C}, g) = f_c(\bar{C}, b) = f_c(C, b) = \bar{P}$ , (ii)  $\tau_c(C, g) = \tau_c(C, b) = g$ , and  $\tau_c(\bar{C}, g) = \tau_c(\bar{C}, b) = b$ . The requirement  $f_c(\bar{C}, g) = f_c(\bar{C}, b) = \bar{P}$  ensures that an active agent can always avoid production by choosing  $\bar{C}$ . In turn, the requirement  $f_c(C, b) = \bar{P}$  allows an active agent to punish a passive agent with a bad record, while still keeping a good

<sup>2</sup> The existence of dominant regions is important for equilibrium selection in the literature on dynamic coordination games (e.g., [Frankel and Pauzner, 2000](#) and [Burdzy et al., 2001](#)) and in the global games literature (e.g., [Morris and Shin, 2003](#)).

record himself. In the first period, half of the population starts with a good record, while the remaining agents start with no record. Note that labels are short-lived, that is,  $\tau_c$  does not depend on the past actions of an agent, only on his current action. Besides, since the passive agent does not make any choice, he does not have a record in the following period. In fact, in the economy with monitoring, active agents were always passive agents in the previous period; while passive agents were always active agents in the previous period.

**Money** There exists a durable but intrinsically useless and inconvertible object, labeled money. For tractability, we assume that money is indivisible and agents are constrained to hold at most one unit of money at a time. An agent is passive if he holds money, and he is active otherwise. In a meeting, the outcome depends on the action of the active agent. Formally, the outcome is determined by a function  $f_m : \{C, \bar{C}\} \rightarrow \{P, \bar{P}\}$ , where  $P$  denotes production and  $\bar{P}$  denotes no production. In the event of production, we assume that money changes hands with probability one-half, while in the event of no production, money does not change hands. A money mechanism is a function  $f_m$  such that  $f_m(C) = P$  and  $f_m(\bar{C}) = \bar{P}$ . The requirement  $f_m(\bar{C}) = \bar{P}$  ensures that an active agent can always avoid production by choosing  $\bar{C}$ . In the first period, half of the population starts with money, while the remaining agents start with no money.

## 2.1. Equilibrium

For each technology of exchange, we consider an equilibrium where active agents always choose  $C$ , on and off the equilibrium path. We start with the case of monitoring and then look at the case of money.

### 2.1.1. Monitoring

On the equilibrium path, the payoff of an active agent who chooses  $C$  is

$$-c + \beta u - \beta^2 c + \beta^3 u + \dots,$$

while a one-shot deviation to  $\bar{C}$  implies

$$0 + \beta 0 - \beta^2 c + \beta^3 u + \dots$$

A deviation implies no cost of production in the current period (and the acquisition of a bad label), no utility of consumption in the following period (due to the bad label), and a return to the payoff of an active agent on the equilibrium path from the next period on. An active agent does not deviate if and only if

$$\beta u > c,$$

which is always true. Off the equilibrium path, i.e., if an active agent meets a passive agent with a bad label, he strictly prefers to choose  $C$  because he incurs no current cost and still keeps the good label. Note that since production takes place in all meetings, monitoring achieves the first-best.

Our monitoring technology assumes that the label of a passive agent is only based on his action in the previous period. More generally, assume that the label of an agent is a function of his actions in all previous periods, and the actions of all agents with whom he has had a direct or an indirect contact. This information is akin to [Kocherlakota's \(1998\)](#) definition of memory. In this case, a deviation from the equilibrium path can be punished with permanent exclusion from consumption. Thus, an active agent does not deviate iff

$$-c + \beta u - \beta^2 c + \beta^3 u + \dots > 0,$$

i.e.,

$$\beta u > c. \tag{1}$$

This is the same condition as in the case where the record of a passive agent only includes his last period action. Thus, in the benchmark environment, the monitoring technology that we consider cannot be improved upon. In other words, there is no other technology of exchange which fares better than the technology we have chosen.

**Comment** One can interpret our monitoring technology as a form of credit. As in the case of credit transactions, the ability to observe one's past history is critical for exchange to happen in our economy. Admittedly, unlike usual credit transactions, in our setting the payment to the creditor does not come from his original debtor but is instead provided by another agent in the economy. In this sense, we have a sort of multilateral credit arrangement. However, even if one models credit transactions as bilateral arrangements, a monitoring technology would still be necessary. What prevents a debtor from defaulting on his debt is the threat of future exclusion from credit transactions, a threat that is only credible if there exists a technology able to release information about bilateral transactions to third-part agents.

### 2.1.2. Money

Let  $V_1$  be the value function of a passive agent and  $V_0$  be the value function of an active agent. If active agents always choose  $C$  in meetings with passive agents, we have

$$V_1 = u + \frac{1}{2}\beta V_1 + \frac{1}{2}\beta V_0,$$

and

$$V_0 = -c + \frac{1}{2}\beta V_1 + \frac{1}{2}\beta V_0.$$

Consider the expression for  $V_1$ . In a meeting, the passive agent obtains utility  $u$ , becomes an active agent with probability one-half (in case money changes hands), and continues as a passive agent with probability one-half (in case money does not change hands). Consider now the expression for  $V_0$ . In a meeting, the active agent incurs a cost  $c$ , becomes a passive agent with probability one-half (in case money changes hands), and continues as an active agent with probability one-half (in case money does not change hands). It is optimal for an active agent to choose  $C$  iff

$$-c + \frac{1}{2}\beta V_1 + \frac{1}{2}\beta V_0 > \beta V_0.$$

Using the expressions for  $V_0$  and  $V_1$ , that can be rewritten as

$$\beta \frac{u+c}{2} > c. \quad (2)$$

If (2) is satisfied, production takes place in all meetings and money achieves the first-best. However, the region of parameters where money achieves the first best is a strict subset of the region of parameters where monitoring achieves the first-best.

**Comment** The assumption that money is only transferred with probability one-half affects the region of parameters where active agents are willing to produce in exchange for money. In particular, if we assume that money changes hands with probability one, money and monitoring are equilibria in the same region of parameters. In this case, money is still inessential, but weakly so. Naturally, one may wonder why we have chosen terms of trade which reduce the region where money is an equilibrium. As it will be shown, the only reason is that this specification dramatically simplifies the analysis when it comes to the general environment. We will return to this issue in the next section, where we will also discuss how our main result is expected to change under a different specification. One should keep in mind though that assuming sub-optimal terms of trade only makes it more notable that money overcomes monitoring as a coordination device.

## 3. General environment

In the benchmark environment, there is always an autarkic equilibrium where an active agent chooses  $\bar{C}$  because he believes that all other agents will do the same. Thus, a particular coordination of beliefs is necessary if exchange is to take place. In what follows, we extend the benchmark by assuming that the economy experiences different states over time and there are remote states where choosing a particular action is a strictly dominant strategy. This extension allows us to compare money and monitoring in terms of their ability to provide the coordination which is required for exchange to happen.

Formally, assume that in any given period, the economy is in some state  $z \in \mathbb{R}$ . The economy starts at  $z = 0$  and the state changes according to a random process  $z_{t+1} = z_t + \Delta z_t$ , where  $\Delta z_t$  follows a continuous and symmetric probability distribution that is independent of  $z$  and  $t$ , with expected value  $E(\Delta z) = 0$  and variance  $\text{Var}(\Delta z) > 0$ . There also exists a state  $\hat{z} > 0$  such that it is strictly optimal to choose  $C$  if  $z > \hat{z}$ , and it is strictly optimal to choose  $\bar{C}$  if  $z < -\hat{z}$ . Throughout, we think of  $\hat{z}$  as a very large number, and label states where  $|z| > \hat{z}$  as dominant regions. In what follows, we first provide a rationale for the dominant regions. We then characterize the unique equilibrium in the economy with money and the unique equilibrium in the economy with monitoring.

### 3.1. The dominant regions

Dominant regions can result from actions of another agent (say, the “planner”) or from purely technological assumptions.<sup>3</sup>

In the first approach, the state  $z$  captures the willingness of the planner to support or disrupt the technologies for exchange. Consider, first, the economy with money. Assume that money is introduced by the planner and the state  $z$  captures

<sup>3</sup> Araujo and Guimaraes (2014) study coordination in the use of money in a similar environment and also discuss reasons for dominance regions in case money is the medium of exchange.

its commitment to support the use of money as a medium of exchange. In states  $z \in [-\widehat{z}, \widehat{z}]$ , the planner does not interfere in the economy. However, in states  $z > \widehat{z}$ , all agents with money are approached by the planner at the beginning of the period and offered to exchange their unit of money into one unit of a commodity which provides utility  $\gamma$ , where  $\beta u > \beta \gamma > c$ . In turn, in states  $z < -\widehat{z}$ , all agents who receive money in exchange for producing a good have their money taxed away by the planner. This taxation scheme can be thought of as a form of sales tax. As it will become clear, the assumption that agents are only taxed if they spend their money is important to make sure that, unlike monitoring, money is a permanent record of past production. Consider now the economy with monitoring. Similarly to the economy with money, we assume that the record-keeping technology is introduced and maintained by a planner and that the state  $z$  captures the commitment of the planner with the use of the technology as a facilitator of exchange. As in the economy with money, in states  $z \in [-\widehat{z}, \widehat{z}]$ , the planner does not interfere in the economy. In turn, in states  $z > \widehat{z}$ , all passive agents with a good label are approached by the planner at the beginning of the period and offered one unit of a commodity which provides utility  $\gamma$ , where  $\beta u > \beta \gamma > c$ . Finally, in states  $z < -\widehat{z}$ , the planner does not maintain the record-keeping technology, which is then unable to record the action of the active agent.

The presence of the planner implies that if the state  $z$  is large enough,  $\beta \gamma > c$  ensures that an active agent is willing to produce in exchange for money in the monetary economy, or in exchange for a good label in the monitoring economy. Moreover,  $u > \gamma$  ensures that a passive agent prefers to keep his money in the monetary economy, or keep his good label in the monitoring economy, and use it in a meeting with an active agent.<sup>4</sup> In turn, the presence of the planner implies that if the state  $z$  is small enough, an active agent has no incentive to produce. In the monetary economy, that is because his money will be taxed away, while in the monitoring economy, that is because there will be no record of his action. We are mostly interested in what happens in states  $z \in [-\widehat{z}, \widehat{z}]$ , where the planner does not interfere in the economy. This is why we think of  $\widehat{z}$  as a very large number.

In what follows, we adopt the first approach. Alternatively, dominant regions may arise for purely technological reasons. For instance, in the case of money, the state  $z$  may reflect changes in the environment which increase the costs of using money as a medium of exchange, for instance, in some states it may be costly to carry money to buy goods. This cost would be zero for  $z \geq -\widehat{z}$  but large enough in states  $z < -\widehat{z}$  to make trade unattractive. Likewise, in the case of monitoring, the record-keeping technology might fail to work in some remote states of nature – i.e., for  $z < -\widehat{z}$ . Under this second set of assumptions, there would be remote regions where trade would not occur, but no state  $\widehat{z}$  such that agents would always choose  $C$  if  $z > \widehat{z}$ . Nevertheless, at the end of Section 4, we posit that the main point of the paper still holds in this case.

### 3.2. Coordination with money

The key difference between our environment with money and the one in Araujo and Guimaraes (2014) is that while in the latter agents meet randomly, in ours an agent with money always meets an agent without money.<sup>5</sup> It turns out though that the characterization of the region of parameters where agents always coordinate in the use of money is proved in exactly the same way. In what follows, we provide a sketch of the proof and refer to Araujo and Guimaraes (2014) for a complete proof.

Let  $\varphi(t)$  denote the probability that any state  $z' > z$  is reached at time  $s + t$  and not before, conditional on the agent being in state  $z$  in period  $s$ . In other words,  $\varphi(t)$  is the probability that, if the economy is currently in state  $z$ , it takes  $t$  periods for it to reach some state  $z' > z$ . We have the following result.

**Proposition 1.** *Agents always coordinate in the use of money in states  $z \in [-\widehat{z}, \widehat{z}]$  if*

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \frac{u + c}{2} > c, \quad (3)$$

*and they never do so if the inequality is reversed.*

Note that (3) does not depend on the value of  $\widehat{z}$ . Thus the acceptability of money in the region of states where money is intrinsically useless and inconvertible does not depend on the size of this region. This should make it transparent that from the perspective of an agent, what drives the coordination in the use of money is the belief that money will be accepted in the future, not the fact that it provides some intrinsic utility in remote states.

A sketch of the proof runs as follows. We use an induction argument, where at each step strictly dominated actions are eliminated. We start with the problem of an active agent in a meeting with a passive agent if the current state is large. In states  $z > \widehat{z}$ , all agents with money are ensured a payoff  $\gamma$ , where  $\beta \gamma > c$ . This implies that there exists  $\widehat{z}_m$  large enough such that it is a strictly dominant action to choose  $C$  if  $z > \widehat{z}_m$ . Consider then the problem of an active agent if the current

<sup>4</sup> In other words,  $u > \gamma$  ensures that the planner does not disrupt the exchange process in the economy. It simply provides the incentives for active agents to choose  $C$  irrespective of the behavior of the other agents, when the state  $z$  is sufficiently large.

<sup>5</sup> The assumption that passive agents randomly meet active agents both in the economy with money and in the economy with monitoring makes it transparent that our main result is not driven by the matching technology. We thank two anonymous referees and the associate editor for pointing out the importance of keeping a uniform matching technology across economies.

state is  $\widehat{z}_m$ . Araujo and Guimaraes (2014, Lemma 2) show that the incentives to produce in exchange for money increase in the likelihood that money will be accepted in the future. Thus, the belief that all the other agents are following a cut-off rule at state  $\widehat{z}_m$ <sup>6</sup> is the most pessimistic belief an agent may hold about the acceptability of money. That is, if an active agent is willing to produce in exchange for money under this pessimistic belief, he is also willing to do so under any other belief. This is more so because the agent also has the option to convert money into a desirable commodity in states  $z > \widehat{z}$ .

Araujo and Guimaraes (2014, Lemmas 3 and 4) then show that, if an agent believes that all the other agents are following a cut-off rule at some state  $z$ , then his expected payoff if he follows the same cut-off rule is sufficient to determine the optimal behavior in state  $z$ . Applied to the problem of the agent in state  $\widehat{z}_m$ , this result implies that an active agent strictly prefers to choose  $C$  in state  $\widehat{z}_m$  if

$$-c + \sum_{t=1}^{\infty} \beta^t \varphi(t) \int_{\widehat{z}_m}^{\infty} \left( \frac{1}{2} V_{1,z'} + \frac{1}{2} V_{0,z'} \right) dF(z'|t) > \sum_{t=1}^{\infty} \beta^t \varphi(t) \int_{\widehat{z}_m}^{\infty} V_{0,z'} dF(z'|t),$$

where  $V_{1,z'}$  ( $V_{0,z'}$ ) is the value of one unit (zero units) of money in state  $z'$ . The left hand side is the expected continuation payoff of choosing  $C$  against an agent with money. It is given by the current cost  $c$ , plus the future benefit of producing in the current period. The latter is given by the discounted probability that the first time the economy reaches some state  $z' > \widehat{z}_m$  after period  $s$  happens in period  $s+t$ , which is given by  $\beta^t \varphi(t)$  multiplied by  $\frac{1}{2} V_{1,z'} + \frac{1}{2} V_{0,z'}$ . Remember that there is probability one-half that money changes hands and a probability one-half that money does not change hands. The cdf of  $z'$  is given by  $F(z'|t)$ , i.e., the probability that the state of the economy is below or equal to  $z'$  conditional on the event that the first time the economy reaches some state  $z' > \widehat{z}_m$  after period  $s$  happens in period  $s+t$ . Finally, the right hand side is the future benefit of not holding money. We can rewrite the above inequality as

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \frac{1}{2} \int_{\widehat{z}_m}^{\infty} (V_{1,z'} - V_{0,z'}) dF(z'|t) > c. \tag{4}$$

The value of having money in some state  $z' > \widehat{z}_m$  is given by

$$V_{1,z'} = u + \beta \int_{-\infty}^{+\infty} \left( \frac{1}{2} V_{1,z''} + \frac{1}{2} V_{0,z''} \right) dG(z''|z'),$$

while the value of not having money is

$$V_{0,z'} = -c + \beta \int_{-\infty}^{+\infty} \left( \frac{1}{2} V_{1,z''} + \frac{1}{2} V_{0,z''} \right) dG(z''|z'),$$

where  $G(z''|z')$  is the probability that the current state is  $z''$  conditional on the previous state being  $z'$ . Importantly, even though the integrals in the expressions above are quite complicated objects,  $V_{1,z'} - V_{0,z'}$  is simply  $u + c$ . Thus, substituting the expressions for  $V_{1,z'}$  and  $V_{0,z'}$  in (4), we obtain that an agent strictly prefers to choose  $C$  in state  $\widehat{z}_m$  if

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \frac{1}{2} (u + c) > c,$$

which corresponds to (3).

We can now use continuity in  $z$  to obtain that there exists some  $\epsilon$  such that each active agent strictly prefers to choose  $C$  in all states  $z > \widehat{z}_m - \epsilon$  if he believes that all other agents are following a cut-off rule at  $\widehat{z}_m$ . Since all agents are following this cut-off rule, the most pessimistic belief an agent may hold about the acceptability of money is now given by the belief that all other agents are following a cut-off rule at  $\widehat{z}_m - \epsilon$ . We can then apply exactly the same reasoning above to conclude that if (3) holds, it is strictly optimal for an active agent to choose  $C$  in state  $\widehat{z}_m - \epsilon$ . Proceeding this way, we can iteratively iterate and thus eliminate choosing  $\bar{C}$  as a strictly dominated action until we reach the state  $-\widehat{z}$ . This implies that  $C$  is the only rationalizable action in states  $z \in [-\widehat{z}, \widehat{z}]$ . Finally, an analogous reasoning applies if the reverse of (3) holds. In this case,  $\bar{C}$  is the only rationalizable action for all  $z \leq \widehat{z}$ . This concludes the sketch of our proof.

Note that (3) can be rewritten as

$$\lambda \beta \frac{u + c}{2} > c,$$

where

$$\lambda = \sum_{t=1}^{\infty} \beta^{t-1} \varphi(t).$$

Thus, the only difference between the condition in (3) for a unique equilibrium where agents always coordinate in the use of money in the general economy and the condition in (2) for the existence of a monetary equilibrium in the benchmark economy is the parameter  $\lambda$ . The condition in (2) is derived under the assumption that agents will be able to coordinate

<sup>6</sup> If an agent follows a cut-off rule at state  $\widehat{z}_m$ , he only produces in exchange for money in states  $z \geq \widehat{z}_m$ .

in the use of money and assesses how the primitives of the model affect the incentives for an *individual* to produce in exchange for money. In contrast, the condition in (3) examines how the primitives of the model affects *society's* ability to coordinate in the use of money. Our paper is about the latter effect and, in particular, about comparing it across economies with different technologies of exchange.

**Comment** We argued in the previous section that the main reason for assuming that money changes hands with probability one-half in the event of production is that it greatly simplifies the analysis. This assumption implies that the difference  $V_{1,z'} - V_{0,z'}$  between having and not having money in (4) is given by  $u + c$ , even though the expressions for  $V_{1,z'}$  and  $V_{0,z'}$  are much more complicated than that. We believe that this assumption, although very useful for tractability reasons, is not driving our result: the region of parameters where agents always coordinate in the use of money is likely to expand under the assumption that money changes hands with probability larger than half. Indeed, consider a hypothetical scenario where in the current period, money changes hands with probability  $p > 1/2$  in the event of production. The probability that money changes hands in future periods though is unchanged, equal to  $1/2$  if production takes place. In this case, the expression in (4) becomes

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) p \int_{\widehat{z}_m}^{\infty} (V_{1,z'} - V_{0,z'}) dF(z'|t) > c.$$

but the expressions for  $V_{1,z'}$  and  $V_{0,z'}$  are unchanged, as this modification in the model does not affect future periods. Hence the analogue of (3) is given by

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) p (u + c) > c. \quad (5)$$

The only difference between (3) and (5) is that the latter features  $p$  instead of  $1/2$ . That is, the region of parameters where agents always coordinate in the use of money in states  $z \in [-\widehat{z}, \widehat{z}]$  expands in this modified scenario. Intuitively, the benefits from producing are larger if the probability of receiving money is higher.

### 3.3. Coordination with monitoring

Consider now the problem of an active agent in state  $z \in [-\widehat{z}, \widehat{z}]$  in period  $t$ , under the belief that all active agents in period  $t + 1$  will be willing to choose  $C$  if and only if  $z_{t+1} \in Z$ , where  $Z$  is some set which contains all states  $z > \widehat{z}$ . Remember that active agents strictly prefer to choose  $C$  in states  $z > \widehat{z}$ . If the agent chooses  $C$ , he obtains

$$-c + \beta \int_{-\infty}^{+\infty} \chi_{z' \in Z} u dG(z'|z) + \beta^2 E(V|z), \quad (6)$$

where  $\chi_{z' \in Z} = 1$  if  $z' \in Z$  and  $E(V|z)$  is the expected value of an active agent two periods ahead, conditional on the current state being equal to  $z$ . In turn, if he chooses  $\bar{C}$ , he obtains

$$\beta^2 E(V|z). \quad (7)$$

Observe that, since labels only last for one period, the last term in (6) coincides with (7). Thus, the net value of choosing  $C$  in state  $z$  is given by

$$-c + \beta u \int_{-\infty}^{+\infty} \chi_{z' \in Z} dG(z'|z), \quad (8)$$

and it only depends on the fundamentals and on the set  $Z$ . This implies that active agents' decisions are strategic complements, i.e., if  $Z^A$  and  $Z^B$  are such that  $Z^A \subset Z^B$ , then the incentives to choose  $C$  given  $Z^B$  are strictly larger than the incentives to choose  $C$  given  $Z^A$ .

We can now prove our result. As in the case with money, the proof uses an induction argument, where at each step strictly dominated actions are eliminated. We start with the problem of an active agent in a meeting with a passive agent if the current state is large. In states  $z > \widehat{z}$ , all agents with a good label are ensured a payoff  $\gamma$ , where  $\beta\gamma > c$ . This implies that there exists  $\widehat{z}_c$  large enough such that it is a strictly dominant action to choose  $C$  if  $z > \widehat{z}_c$ . Consider then the problem of an active agent if the current state is  $\widehat{z}_c$ . Since the distribution of  $\Delta z$  is symmetric, the probability the economy will be at a state  $z > \widehat{z}_c$  next period is  $\frac{1}{2}$ . Suppose an agent believes nobody will be willing to produce in the next period if  $z < \widehat{z}_c$ . Since active agents' decisions are strategic complements, this is the worst scenario to choose  $C$ . Using (8), it is optimal to produce at  $\widehat{z}_c$  if

$$-c + \frac{1}{2} \beta u > 0.$$

As in the case with money the last part of the proof uses continuity on  $z$  to obtain that there exists some  $\epsilon$  such that active agents strictly prefer to choose  $C$  in all states  $z > \widehat{z}_c - \epsilon$ . In consequence, once choosing  $\bar{C}$  has been ruled out for all  $z \geq \widehat{z}_c$ , the choice  $C$  becomes a strictly dominant action for an agent in any state  $z > \widehat{z}_c - \epsilon$ . But then if  $-c + \frac{1}{2} \beta u > 0$ , an

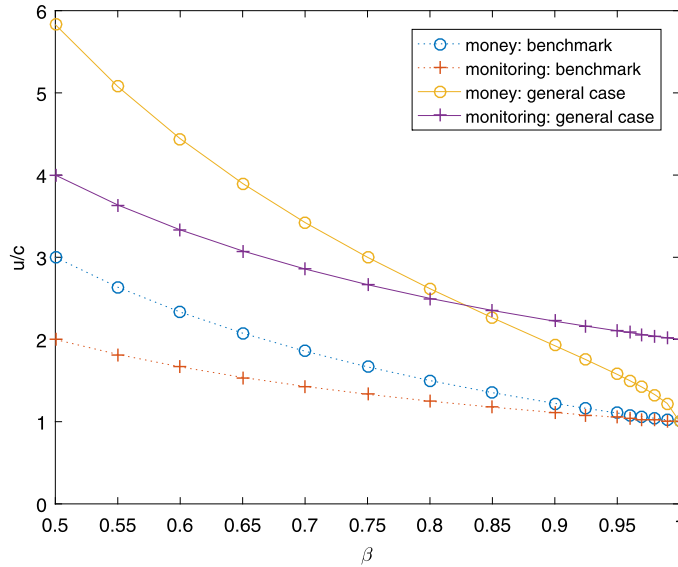


Fig. 1. Money and monitoring in the general and benchmark environments.

active agent finds it strictly optimal to choose  $C$  in state  $z = \hat{z}_c - \epsilon$ . Proceeding this way, we can iteratively iterate and thus eliminate choosing  $\bar{C}$  as a strictly dominated action until we reach the state  $-\hat{z}$ . This implies that  $C$  is the only rationalizable action in states  $z \in [-\hat{z}, \hat{z}]$ . An analogous reasoning applies if  $-c + \frac{1}{2}\beta u < 0$ . In this case,  $\bar{C}$  is the only rationalizable action for all  $z \leq \hat{z}$ . Proposition 1 summarizes our result.

**Proposition 2.** Agents always coordinate in the use of monitoring in states  $z \in [-\hat{z}, \hat{z}]$  if

$$\frac{\beta u}{2} > c, \tag{9}$$

and they never do so if the inequality is reversed.

Note that, as in the case with money, the value of  $\hat{z}$  does not enter in (9). In other words, the existence of dominant regions affects agents' behavior in  $[-\hat{z}, \hat{z}]$  but it does not matter how remote these regions are. In turn, as in the case with money, the difference between (1) and (9) reflects the impact of coordination. While (1) describes how the fundamentals affect the individual agent decision taking as given that all other agents will coordinate in the use of monitoring, (9) describes how fundamentals affect society's ability to coordinate in the use of monitoring.

#### 4. Money and monitoring

In the benchmark environment money was inessential because we assumed away coordination problems. In the general environment, however, agents always coordinate in the use of money in a region of parameters where they cannot coordinate in the use of monitoring. Formally, combining Propositions 1 and 2, we obtain that agents coordinate in the use of money but do not coordinate in the use of monitoring iff

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \frac{1}{2} (u + c) > c > \frac{1}{2} \beta u.$$

Since  $\sum_{t=1}^{\infty} \beta^t \varphi(t) \left(\frac{u+c}{2}\right)$  converges to  $\frac{u+c}{2}$  when  $\beta$  converges to one,<sup>7</sup> there exists  $\beta^*$  such that, for all  $\beta > \beta^*$ , agents can coordinate in the use of money but they cannot coordinate in the use of monitoring. Fig. 1 shows the regions of parameters in which money and monitoring are an equilibrium in the benchmark and in the general environment.<sup>8</sup> The dotted curves depict the condition for existence of equilibrium in the benchmark model and the solid curves show the condition for coordination in the general environment. The cases with money are depicted with circles and the cases with monitoring are

<sup>7</sup> Remember that  $\varphi(t)$  denotes the probability that any state  $z' > z$  is reached at time  $s + t$  and not before, conditional on the agent being in state  $z$  in period  $s$ . When  $\beta$  converges to one  $\sum_{t=1}^{\infty} \beta^t \varphi(t)$  converges to  $\sum_{t=1}^{\infty} \varphi(t)$ . Since  $z_t$  follows a random walk it must be the case that, with probability one, the economy will eventually reach some state  $z' > z$ , i.e., it must be the case that  $\sum_{t=1}^{\infty} \varphi(t) = 1$ .

<sup>8</sup> We consider a normal distribution for  $\Delta z$  with  $E(\Delta z) = 0$ . The probabilities  $\varphi(t)$  are obtained from Monte Carlo simulations (they do not depend on the variance  $\text{Var}(\Delta z)$ ).



depicted with asterisks. In the benchmark model, autarky is the unique equilibrium in the region below the dotted curves, and there are multiple equilibria above the dotted curves. In the general environment, agents always coordinate in the use of the medium of exchange above the solid curves and never do so below the solid curves.

Patient agents are willing to produce in exchange for money as long as they believe that they will be able to eventually spend the money. That is, although the belief that money may be accepted in the near future matters, acceptability in later periods is also important. This makes it easier to coordinate in the use of money. Intuitively, if an agent believes that eventually he will find someone that accepts money, he is willing to produce in exchange for money even if he believes that a large number of his future partners will not accept money. In contrast to money, since labels only last for one period, coordination in the use of monitoring critically depends on the belief that the good label will be “spent” in the near future. As a result, agents can only coordinate in the use of monitoring if the primitives of the economy compensate for the belief that, while the disutility of production is certain, the benefit of acquiring the good label is short lived.

Lastly, it turns out that patience is critically important in our setting because we have considered a benchmark environment where monitoring is an equilibrium in a region of parameters that is strictly larger than money. We have chosen this benchmark mostly for tractability reasons. Besides, it makes it transparent that, even if money is strictly worse than monitoring absent coordination frictions, it can become strictly better than monitoring when coordination frictions take center stage. In [Appendix A](#), we consider a modification of our benchmark environment where money and monitoring are equilibria in the same region of parameters. We obtain that money strictly dominates monitoring in the general environment for all  $\beta > 0$ . This result reinforces our message that money is essential for coordination reasons, as it allows to dispense with the restriction that agents must be relatively patient. However, it remains true that the superiority of money over monitoring as a coordinating device is stronger, the higher the discount factor.<sup>9</sup>

**Comment** Consider the case where technological reasons alone may disrupt exchange in some states of the world, i.e., there exists a state  $-\hat{z}$  such that the cost of using money (respectively, the cost of maintaining the record-keeping technology) is zero in all states  $z \geq -\hat{z}$ , but it is large enough to prevent trade in states  $z < -\hat{z}$ . In other words, consider the scenario where we removed the assumption that agents always choose  $C$  if  $z > \hat{z}$ . In this case, autarky is always an equilibrium. However, [Propositions 1 and 2](#) imply that autarky is the unique equilibrium when  $u/c$  is below the respective solid line in [Fig. 1](#). Hence, when  $\beta$  is large, there is a range of values for  $u/c$  in which money is essential because agents may be able to coordinate in its use but they can never coordinate in the use of monitoring. That is, as long as we are willing to assume the existence of states where production will not be possible or desirable for the active agent, for large values of  $\beta$ , it is easier to coordinate in the use of money.

## 5. Final remarks

We showed that if agents are patient, the region of parameters where they coordinate in the use of money is strictly larger than the region of parameters where they coordinate in the use of monitoring. Thus money is essential because it overcomes coordination frictions.

In order to render our model tractable, we made some assumptions that deserve discussion. First, we assumed that money is indivisible and that agents can hold at most one unit of money at a time. This assumption follows earlier models of money and search, i.e., [Kiyotaki and Wright \(1989, 1993\)](#), [Trejos and Wright \(1995\)](#), and [Shi \(1995\)](#). [Lagos and Wright \(2005\)](#) consider a model where agents participate in a centralized Walrasian market in every period, which allows to introduce divisible money in a tractable way. The difficulty with introducing divisible money is that it breaks the simple dichotomy between accepting money and not accepting money which greatly simplified our analysis. Besides, the introduction of periodic access to a centralized Walrasian market has non-trivial implications on coordination, and it is not clear from the outset whether it constitutes a better benchmark than the one considered here, with fully decentralized trade. Finally, what our results show is that the key feature of money that facilitates coordination in its use is the fact that it is a permanent record of past transactions, a feature that does not depend on its divisibility.

We assumed that monitoring is sustained by a monitoring technology which only records the last period action of an agent.<sup>10</sup> One implication is that the current payoff of a passive agent depends on his history, but the optimal action of an active agent does not depend on past events. The analysis would become much more involving if an active agent anticipates that his current action will impact his optimal action the next time he is called upon to choose again. However, the choice of a one-period monitoring technology cannot be defended only on grounds of tractability, as it matters for our results. Indeed, the difficulty to coordinate on monitoring comes exactly from the fact that labels are short-lived. The way in which we attempted to overcome this issue is by considering a benchmark where the inessentiality of money does not depend on the amount of information the monitoring technology is able to record. That is, without coordination frictions, one-period monitoring suffices. This makes it clear that increasing the amount of information in the record-keeping technology may only be meaningful for coordination reasons, thus reinforcing the message that agents are better able to coordinate on money because money is a permanent record of past transactions.

<sup>9</sup> We are thankful to both referees for pointing out this issue, and we are thankful for their suggestions on how to deal with it.

<sup>10</sup> Our assumptions that records are shortly-lived resemble modern credit laws. For instance, bankruptcy can only appear on credit reports for 7 years. We thank an anonymous referee for pointing this out.

## Appendix A

In what follows, we modify the basic environment by considering a case where money and monitoring are equilibria in the same region of parameters. To do so, we kept the same environment in the economy with money but we changed the environment with monitoring as follows. We assume that, if an active agent chooses  $C$  in his meeting with a passive agent, he acquires the good label with probability  $\rho \in [0, 1]$ . In the original environment  $\rho = 1$ . Under this modification, on the equilibrium path, the payoff of an active agent who chooses  $C$  is

$$-c + \beta\rho u - \beta^2\rho c + \beta^3\rho u + \dots$$

In words, the agent incurs the cost  $c$  and, in the following period, acquires the good label with probability  $\rho$ , in which case he receives the good. He then becomes a passive agent once more, and the process repeats itself. Note that an active agent only expects to incur the production cost with probability  $\rho$ , the probability of meeting an agent with a good label. In turn, a one-shot deviation to  $\bar{C}$  implies

$$0 + \beta 0 - \beta^2\rho c + \beta^3\rho u + \dots$$

An active agent does not deviate if and only if

$$\beta\rho u > c.$$

Off the equilibrium path, i.e., if an active agent meets a passive agent with a bad label, he strictly prefers to choose  $C$  because he incurs no current cost and still keeps the good label with a positive probability. In the basic environment with money, the analysis is the same as in the previous version of the paper, that is, an active agent strictly prefers to produce to a passive agent if and only if

$$\beta \frac{u+c}{2} > c.$$

Thus, we obtain that money and monitoring are equilibria in the same region of parameters if and only if

$$\rho = \frac{u+c}{2u},$$

which we henceforth assume. The exercise we have in mind is considering how money fares compared to monitoring in the general environment, given that, in the basic environment, both mechanisms of exchange are equilibria in the same region of parameters. In particular, we want to examine the referee's conjecture that the superiority of monitoring over money in the general environment if agents are relatively impatient is only a consequence of the fact that money is inherently worse than monitoring.

We now turn to the general environment. The analysis in the environment with money is exactly the same as in the previous version of the paper, that is, agents always coordinate in the use of money in states  $z \in [-\hat{z}, \hat{z}]$  if

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \frac{u+c}{2} > c, \quad (10)$$

and they never do so if the inequality is reversed. Remember that  $\varphi(t)$  denotes the probability that any state  $z' > z$  is reached at time  $s+t$  and not before, conditional on the agent being in state  $z$  in period  $s$ . In other words,  $\varphi(t)$  is the probability that, if the economy is currently in state  $z$ , it takes  $t$  periods for it to reach some state  $z' > z$ .

Consider now the environment with monitoring. We start with the problem of an active agent in a meeting with a passive agent if the current state is large. Remember that, in states  $z > \hat{z}$ , all agents with a good label are ensured a payoff  $\gamma$ , where we assume that  $\beta\rho\gamma > c$ . This implies that there exists  $\hat{z}_c$  large enough such that it is a strictly dominant action to choose  $C$  if  $z > \hat{z}_c$ . Consider then the problem of an active agent if the current state is  $\hat{z}_c$ . Since the distribution of  $\Delta z$  is symmetric, the probability the economy will be at a state  $z > \hat{z}_c$  next period is  $\frac{1}{2}$ . Suppose an agent believes nobody will be willing to produce in the next period if  $z < \hat{z}_c$ . Since active agents' decisions are strategic complements, this is the worst scenario to choose  $C$ . It is optimal to produce at  $\hat{z}_c$  if

$$\frac{1}{2}\beta\rho u > c. \quad (11)$$

As we did in the previous version of the paper, the proof now uses continuity on  $z$  and the iterative elimination of strictly dominated strategies to obtain that agents always coordinate in the use of monitoring in states  $z \in [-\hat{z}, \hat{z}]$  if (2) holds, and they never do so if the inequality is reversed.

Finally, we compare (1) and (2). Note that, since  $\varphi(1) = \frac{1}{2}$ , a sufficient condition for the left-hand side of (1) to be strictly larger than the left-hand side of (2) is that

$$\frac{1}{4}(u+c) \geq \frac{1}{2}\rho u,$$

which is satisfied at equality when  $\rho = \frac{u+c}{2u}$ . Thus, as conjectured by the referee, if money and monitoring are equilibria in the same region of parameters in the basic environment, then money strictly dominates monitoring in the general environment for all  $\beta > 0$ .

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